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B.Sc. (Hons) Part - I, Paper - I

Rank of Matrix (Remaining part)

Equivalent Matrices:

If a matrix B is obtained from A by a finite no. of elementary transformations to A.
B is said to be equivalent to A.
Denoted by $B \sim A$.

Also,

(i) $A \sim A$,

(ii) $B \sim A \Rightarrow A \sim B$

(iii) $A \sim B,\ B \sim C \Rightarrow A \sim C$

Elementary matrices:

A matrix obtained from a unit matrix by some elementary row or column transformations is called an elementary row or column matrix.
Denoted by E-matrix.

⇒ All elementary matrices are non-singular

⇒ Rank of a matrix does not change after elementary transformations.

⇒ Normal form of a matrix

$$\begin{bmatrix} I_n & 0 \\ 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} I_n & 0 \end{bmatrix}, \quad \begin{bmatrix} I_n \\ 0 \end{bmatrix}, \quad \begin{bmatrix} I_n \end{bmatrix}$$

where I_n is the n-rowed unit matrix.

$$\begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$$ First canonical form:

⇒ Every non-zero matrix of rank r can be reduce to any of normal forms.

Example. 1. Find the rank of

$$A = \begin{bmatrix} 1 & -1 & 3 & 6 \\ 1 & 3 & -3 & -4 \\ 5 & 3 & 3 & 11 \end{bmatrix}$$

Soln -

$$A \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 4 & -6 & -10 \\ 5 & 8 & -12 & -19 \end{bmatrix} \quad \begin{array}{l} C_2 \to C_2 + C_1 \\ C_3 \to C_3 - 3C_1 \\ C_4 \to C_4 - 6C_1 \end{array}$$

$$\Rightarrow A \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 4 & -6 & -10 \\ 0 & 8 & -12 & -19 \end{bmatrix} \quad \begin{array}{l} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - 5R_1 \end{array}$$

$$\Rightarrow A \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 10 \\ 0 & 2 & 2 & 19 \end{bmatrix} \quad \begin{array}{l} C_2 \to \frac{1}{4}C_2 \\ C_3 \to -\frac{1}{6}C_3 \\ C_4 \to -C_4 \end{array}$$

~~$R_3 \to$~~

→

$$\Rightarrow \quad A \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \quad \begin{array}{l} C_3 \to C_3 - C_2 \\ C_4 \to C_4 - 10C_2 \\ R_3 \to R_3 - 2R_2 \end{array}$$

$$\Rightarrow \quad A \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix} \quad C_4 \leftrightarrow C_3$$

$$\Rightarrow \quad A \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad C_3 \to (-1) \times C_3$$

$$\Rightarrow \quad A \sim \begin{bmatrix} I_3 & 0 \end{bmatrix}$$

So, the rank of A is 3.